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In quest of "just" the Standard Model on D-branes at a singularity

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Abstract: In this note we explore the possibility of obtaining gauge bosons and fermionic spectrum as close as possible to the Standard Model content, by placing D3-branes at a $\mathbb{Z}_N$ orbifold-like singularity in the presence of D7-branes. Indeed, we find that this is plausible provided a sufficiently high N is allowed for and the singular point is also fixed by an orientifold action. If extra charged matter is not permitted then the singularity should necessarily be non-supersymmetric. Correct hypercharge assignments require a dependence on some abelian gauge D7-groups. In achieving such a construction we follow a recent observation made in ref. [1] about the possibility that, the three left handed quarks, would present different U(2) transformation properties.

Keywords: D-branes, Superstring Vacua, Standard Model.

We encode under the name of *String phenomenology* [2] the different attempts to embed the Standard Model of fundamental interactions, or plausible extensions of it, into the framework of string theory. Activity in this area started in the middle eighties, especially in the so called *perturbative* heterotic string context, and many features have been understood since then. A lesson to recall is that, in spite of the enormous degeneracy of $D = 4$ dimensional string vacua, leading to loss of predictability, not everything can be fitted into such a context. String theory imposes severe constraints indeed on model building. A neat example is provided, for instance, by the fact that, in heterotic string theory, the contribution to the mass of a state in a given representation of the gauge group is proportional to the dimension of the representation. Thus, high dimensional massless representations are not allowed for in perturbative heterotic string models.[1]

Stringy constraints, related to the structure of anomaly cancellation, can also be found behind the *failure* to build exactly the Standard Model (SM) in a string theory framework. Despite the many models constructed with gauge and massless fermionic sector quite close to the SM one, such models generically possess extra visible fermionic matter. This is valid for heterotic perturbative constructions and also for other string constructions involving, for instance, type-II D-branes.

This may seem rather surprising since Standard Model, $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$, generations already produce anomaly free combinations. String consistency requirements, like modular invariance in perturbative heterotic string or tadpole cancellation in open string models, certainly imply that such models are free of anomalies. However, we should notice that stringy constraints are generically stronger than anomaly cancellation conditions. These stronger requirements often manifest in the presence of extra gauge factors, and thus, of extra chiral fermions which must be generically present for canceling their anomalies. For instance, in D-brane models, U(2) unitary groups appear, rather than SU(2). As a consequence, doublets (2_1) or anti-doublets 2_{-1} are distinguished by their different U(1) charges (indicated as 1 and -1 subscripts respectively) and tadpole cancellation requires the same number of both of them.

Hence, if the three left handed quark generations were just mere replications of each other, say $3(3, 2)$, then 9 anti-doublets should also be present. Given that three of them can be identified with SM leptons, still six extra doublets will be required by stringy considerations. Nevertheless, if two left quarks were doublets (or anti-doublets) and the other one was an U(2) anti-doublet (doublet) then no extra doublets would be needed.

It is by noticing this fact that, only very recently [1], string models with "just" the, non supersymmetric, SM gauge and fermionic content have been obtained. Such constructions were achieved by considering type-IIA D6-branes wrapping at angles on a six dimensional torus in the presence of orientifold planes and NS background fields. The relevant observation there is thus that quark doublets generations behave differently under U(2) transformations.

We should notice that many *Standard like models* found in the literature contain, apart from extra doublets, extra vector like triplets in the spectrum. The origin of these triplets

[1] This appears to be an important limitation to GUT like models [3].

is somewhat different and is related to the singularity structure. For instance, they usually appear due to "mod N" identifications in models with $\mathbb{Z}_N$ like singularities (see for instance D3-brane models in [4, 5, 6]). Once an extra triplet appears, apart from those of the SM, then a corresponding U(3) anti-triplet is again required for tadpole cancellation to occur. Interestingly enough, for D3-brane models at $\mathbb{Z}_N$ singularities, $\mathbb{Z}_3$ like singularities are the only supersymmetric ones leading to 3 (equivalent) generations [4] and they always lead to vector like triplets.

In this note we attempt, by invoking similar arguments as in ref. [1] related to anomaly cancellation, to build models as close as possible to the Standard Model, in the framework of configurations of D3-branes stuck on a $\mathbb{R}^6/\mathbb{Z}_N$ and in the presence of D7- branes. By *"as close as possible"* we mean the Standard Model minimal content of 3 generations of quarks and leptons, without extra massless matter like, for example, SU(2) doublets or SU(3) triplets.

In order for the spectrum to contain left quarks doublets as well as anti-doublets of U(2), the singularity point is also required to be invariant under an orientation reversal (orientifold) action. Moreover, in order to avoid extra charged matter, due to orbifold-orientifold identifications, non supersymmetric singularities with high values of N must be considered.

Our analysis should be viewed as a first step, in the spirit of a bottom-up approach (see for instance [4]), in the construction of a full string model.

In fact, this partial structure should be further embedded into a globally consistent string model. Depending on the features of the singularity, this could be achieved, for instance, by considering type-IIB orientifold (a cristalographic singularity) or generically F-theory compactifications [4, 7]. It is important to notice that many phenomenological features will depend on the global structure.

An relevant difference with respect to the approach in [4], that should become clear from discussion below is that, in our proposal, hypercharge necessarily involves U(1) generators coming from 77 branes sectors if correct hypercharge (free of anomalies) Standard model assignments are looked for. This means that such groups must be gauged and not merely global symmetries. Therefore, generically, states charged with respect to D7 groups will carry hypercharge. For the full construction to be consistent it should be ensured that hypercharge anomalies carried by D7 states could be canceled or that, such states, could be finally projected out.

We will not address this second step here, involving the full construction of the D7 brane sector. In this sense, our construction implies the identification of *necessary* conditions for plausibly having just the Standard Model content in this context. Nevertheless, we will argue that there seems to be enough freedom in the 77 brane sector for achieving full consistency.

An extensive treatment of D3-branes at singularities was presented in [4]. We will closely follow the notation used there and borrow some of the results. Let us recall some facts.

The states corresponding to a set of n D3-branes stuck at a $\mathbb{R}^6/\mathbb{Z}_N$ singularity are obtained by keeping original fermionic and bosonic states invariant under the action of $\mathbb{Z}_N$

generator θ. Recall that θ rotates string coordinates as well as Chan-Paton indices. The latter can be achieved by a general twist matrix given by

$$\gamma_{\theta,3} = \mathrm{diag}\left(\mathrm{I}_{n_0}, e^{2\pi i/N}\mathrm{I}_{n_1}, \ldots, e^{2\pi i(N-1)/N}\mathrm{I}_{n_{N-1}}\right) \tag{1}$$

where I_{n_i} is the $n_i \times n_i$ unit matrix, and $\sum_i n_i = n$. Twist information can be encoded into the vector

$$\mathbf{V}_3 = \frac{1}{N}(0, \ldots, 0, 1, \ldots, 1, \ldots, (N-1), \ldots, (N-1)) \tag{2}$$

with n_0 0's, n_1 1's etc.

For instance, the four fermionic states on the D3-brane world-volume, are described by Ramond states $\lambda|s_1, s_2, s_3, s_4\rangle$, with $s_i = \pm 1/2$ and $\sum_i s_i = \mathrm{odd}$ where λ is a Chan-Paton factor. By convention we choose $s_4 = -1/2$ to be left-handed fermions. $\mathbb{Z}_N$ rotation on Fock string states can be encoded in the vector (a_1, a_2, a_3, a_4) with $a_1 + a_2 + a_3 + a_4 = 0 \bmod N$, and it is represented by $\mathcal{R}(\theta)|s_1, s_2, s_3, s_4\rangle = e^{2\pi i a_\alpha s_\alpha/N}|s_1, s_2, s_3, s_4\rangle$. Invariant fermionic states are given by

$$\lambda = e^{2\pi i a_\alpha/N}\gamma_{\theta,3}\lambda\gamma_{\theta,3}^{-1}\,. \tag{3}$$

Similarly, the action on NS states, namely, gauge bosons $\lambda\psi^{\mu}_{-1/2}|0\rangle$, with μ along the D3-brane, or complex scalars given by $\lambda\Psi^{r}_{-1/2}|0\rangle$ (with $r = 1, 2, 3$ labeling a complex plane transverse to the D3-brane) can be encoded in a vector $(b_1, b_2, b_3, 0)$ with $b_1 = a_2 + a_3$, $b_2 = a_1 + a_3$, $b_3 = a_1 + a_2$, where we have included a fourth space-time world volume coordinate with $b_4 = 0$. Thus, invariant NS states are given by

$$\lambda = e^{-2\pi i b_r/N}\ \gamma_{\theta,3}\ \lambda\ \gamma_{\theta,3}^{-1}\,. \tag{4}$$

Spectra can be easily computed when Chan-Paton terms are written in a Cartan Weyl basis (details can be found in[8, 9]). Namely, CP generators are organized into Cartan algebra generators $\lambda_I = H_I$, $I = 1, \ldots, n$ while charged generators are labeled by $\mathrm{U}(n)$ root vectors $\rho_3 = (\underline{1}, -1, 0, \ldots, 0)$ where underlining indicates all possible permutations.

Thus, (3) and (4) select charged generators satisfying

$$\begin{aligned} \rho_3 \cdot V_3 &= \frac{-a_\alpha}{N} \bmod \mathbb{Z} \\ \rho_3 \cdot V_3 &= \frac{b_r}{N} \bmod \mathbb{Z} \end{aligned} \tag{5}$$

for fermionic and NS states respectively. Cartan generators are projected out whenever phases are non vanishing. The resulting spectrum in the 33 sector reads

$$\begin{aligned} &\text{Vectors} && \prod_{i=0}^{N-1} \mathrm{U}(n_i) \\ &\text{Complex Scalars} && \sum_{r=1}^{3}\sum_{i=0}^{N-1}(n_i, \overline{n}_{i-b_r}) \\ &\text{Fermions} && \sum_{\alpha=1}^{4}\sum_{i=0}^{N-1}(n_i, \overline{n}_{i+a_\alpha}) \end{aligned} \tag{6}$$

where sub-indices are understood modulo N. Also, fundamental (anti-fundamental) representations of SU(n) carry unit (-1) charge with respect to the U(1) factor in U(n).

D7-branes are generically required in order to achieve cancellation of RR charges. Take, for instance, 7_3 branes, orthogonal to complex coordinate Y_3 and containing a set of D3-branes as considered above and choose b_3 = even. The Chan-Paton embedding can be defined as

$$\gamma_{\theta,7_3} = \mathrm{diag}\left(\mathrm{I}_{u_0}, e^{2\pi i/N}\mathrm{I}_{u_1}, \ldots, e^{2\pi i(N-1)/N}\mathrm{I}_{u_{N-1}}\right) \tag{7}$$

with $\sum_i u_i = u$ and a corresponding *shift* vector $\mathbf{V}_{7_3}$ can be assigned as in eq. (2). The massless $\mathbf{37_3}$ spectrum is then found from the conditions

$$\begin{aligned} \rho^{37_3}\cdot V_{37_3} &= \frac{-1/2b_3}{N} \bmod \mathbb{Z} \\ \rho^{37_3}\cdot V_{37_3} &= \frac{(b_1+b_2)/2}{N} \bmod \mathbb{Z} \end{aligned} \tag{8}$$

for left handed fermions and scalars respectively. Here $V_{37_3} = (V_3, V_{7_3})$ and $\rho_{37_3} = (\underline{1,0\ldots0}; \underline{-1,0\ldots0})$ are $n+u$ dimensional vectors.

The resulting 37+73 spectrum is

$$\begin{aligned} &\text{Fermions} && \sum_{i=0}^{N-1}\left[\left(n_i, \overline{u}_{i+\frac{1}{2}b_3}\right) + \left(u_i, \overline{n}_{i+\frac{1}{2}b_3}\right)\right] \\ &\text{Complex Scalars} && \sum_{i=0}^{N-1}\left[\left(n_i, \overline{u}_{i-\frac{1}{2}(b_1+b_2)}\right) + \left(u_i, \overline{n}_{i-\frac{1}{2}(b_1+b_2)}\right)\right]. \end{aligned} \tag{9}$$

Similar results are obtained for other D7_r-branes, transverse to the r^{th} complex plane, just by replacing $b_3 \to b_r$ etc.

Here we are concentrating in sectors containing the n D3 at the singularity, since we pretend to place the SM on them. However, we must also take D7 branes into account. When non-compact configurations are considered D7 branes are non dynamical and the corresponding 77 groups are global symmetries. Nevertheless, when the above configuration is embedded in a compact space, 77 sector must be treated in an equal footing with the others. We will not address this computation here but we will comment on it below.

The above Chan-Paton twists, though consistent with a $\mathbb{Z}_N$ action, must be further constrained in order to ensure twisted RR fields charge cancellation. As is well known [11, 12, 4, 10] for generic n_i, ${u^r}_i$ these are equivalent to non-abelian SU(n_i) gauge anomaly cancellation. Namely,

$$\sum_{\alpha=1}^{4}(n_{i+a_\alpha} - n_{i-a_\alpha}) + \sum_{r=1}^{3}\left(u^r_{i+\frac{1}{2}b_r} - u^r_{i-\frac{1}{2}b_r}\right) = 0 \tag{10}$$

Notice (see (6–9)) that the term with a positive (negative) sign is the multiplicity of the $\mathbf{n}_i$ fundamental (anti-fundamental) representation of SU(n_i). Thus, the same number of fundamental and anti-fundamental SU(n) representations must be present in the spectrum. Equivalently, another way to read the above result is that the net U(1) charge, for each U(n), must vanish since a fundamental representation carries charge 1 while anti-fundamental -1 . Interestingly enough we find constraints even when n_i take specific values like $n_i = 0, 1, 2$ which would lead, respectively to, no gauge group at all or U(1) or U(2) where non-abelian anomalies are not expected.

Recall that, since only fermions in bi-fundamental representations of the form $(n_i, \overline{n}_{i+a_\alpha})$ appear in the 33 spectrum, we could have $(3, \overline{2})$ left quarks (here $\overline{2} = 2_{-1}$) which are U(2) anti-doublets (or $(\overline{3}, 2)$) but not $(3, 2)$ doublets. Therefore, if we managed to obtain 3 generations of left handed quarks, we would *always* need six extra doublets as we have discussed above.[2]

The possibility of having different U(2) transformations for left quark generations opens up when $\mathbb{Z}_N$ singularity is placed onto an orientifold plane. Indeed, orientifold identifications allow for the presence of (n, n) and $(\bar{n}, \bar{n})$ as well.

Invariance under an orientifold action Ω imposes further constraints on twists considered above [8]. In particular $\gamma_{\theta,3} = (\tilde{\gamma}_{\theta,3}, \tilde{\gamma}^*_{\theta,3})$. This twist leads to a replicated group with a spectrum invariant under conjugation of representations. Thus we see that ranks $n_j = n_{-j}$ and factors U(n_j) and U(n_{-j}) are exchanged and must be identified in the quotient by Ω. Similarly the fundamental representation n_j goes over to the anti-fundamental representation $\overline{n}_j$, and vice-versa. When the two entries of some bi-fundamental are charged with respect to the same group in the quotient, the antisymmetric combination must be kept etc. Again, an operative way to easily compute the spectrum is to work in a Cartan-Weyl basis. In particular, equations (5, 8) are still valid if a shift vector

$$\mathbf{V}_3 = \frac{1}{N}(0, \dots, 0, 1, \dots, 1, \dots, P \dots, P)\,, \tag{11}$$

with $N = 2P(2P+1)$, is assigned to $\tilde{\gamma}_3$ (and similarly a $\mathbf{V}_{7_r}$ to $\tilde{\gamma}_{7_3}$) and by replacing + and − signs by ± 1 in root vectors.

For instance $\rho_{33} = (\underline{\pm 1, \pm 1, 0 \dots, 0})$ should be used. Notice that these correspond to $SO(2n)$ charged generator roots, as expected, after orientifold projection. We will not write down the generic spectrum but rather concentrate on specific examples in order to illustrate our discussion.

Let us look for an explicit realization of the above ideas. Namely, we search for SM gauge group on D3 branes at the ZN orientifold singularity, in the presence of D7 branes, with a basic structure of 3 left handed SM SU(3) × SU(2) × U(1)$_Y$ quarks as

$$3\left(3, 2, \frac{1}{6}\right) = 2\left(3, \bar{2}, \frac{1}{6}\right)_{1,-1} + \left(3, 2, \frac{1}{6}\right)_{(1,1)}. \tag{12}$$

Subscripts indicate the charges corresponding to U(1) factors in U(3) × U(2) .

Let us first notice that it is natural to place both SU(3) and SU(2) gauge groups on D3-branes. In fact, as can be seen already from the spectrum in eq. (9), multiplicity of 7_r3 states is just one (due to second constraint in (8)) and therefore, it is not possible to get 3 left handed quark generations, for instance, by placing SU(2) on a 77 sector. Since, as can

[2] An alternative way, which deserves further investigation, to cancel extra doublets anomalies could be achieved by turning on B and F fluxes in the lines recently suggested in ref [13].

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be seen from eq. (5), fermion multiplicity is given by the number equal twist eigenvalues, we must have a twist action on fermions of the form

$$(a_1, a_2, a_3, a_4) = (a, a, b, c) \tag{13}$$

with $c = -(b + 2a) \bmod N$. Moreover, $b \neq a \neq c \bmod N$ in order to avoid three identical generations and also $b \neq c \bmod N$.

A $(3, \overline{2})$ is represented by a 33 root vector $= (\underline{1, 0, 0}, \underline{-1, 0}, \ldots)$, while a $(3, 2)$ corresponds to $= (\underline{+, 0, 0}, \underline{+, 0}, \ldots)$, where the first three entries correspond to SU(3) and the two others to SU(2).

Thus, we are lead to CP twists of the form

$$V = \frac{1}{N}\left(-\frac{a+b}{2}, -\frac{a+b}{2}, -\frac{a+b}{2}, \frac{a-b}{2}, \frac{a-b}{2}, d_1, d_2, \ldots\right). \tag{14}$$

It produces the desired states in eq. (12) if hypercharge is defined as $Y = \frac{Q_3}{6} + 0\frac{Q_2}{6} + \cdots$

Entries d_1, d_2 etc. allow for the presence of extra U(1) factors that could be needed to accommodate the rest of the SM spectrum (one such factor is added in model below).

We have stressed that Q_2 can not be part of hypercharge since U(2) doublets and anti-doublets, carrying opposite such charge, must give the same 1/6 hypercharge contribution. This would, necessarily, lead to include U(1) charges originated in D7-brane groups in the definition of Y. The reason is that, as it can be checked, it is not possible to accommodate all right quarks and left leptons in 33 sector for any N. Some of them must necessarily come from 37_r sectors. Since Q_3 normalization is already fixed in order to produce correct left quark assignements and Q_2 is not present in Y correct charges for such states must include U(1) generators from 77 sectors (otherwise right quarks from 37 sectors would carry $Y = -1/6$ and/or leptons $Y = 0$ hypercharge).

Therefore D7-branes must be embedded in a global compact manifold for the groups to become gauged. This differs from the approach in [4], where hypercharge appeared as the diagonal combination of D3-brane U(1) groups.

Notice that $b = -2a \pmod N$ would lead to a supersymmetric singularity. However, for the twist above, this value would produce extra $(3, 1, -1/3)$ representations (coming from conjugate antisymmetric representations of SU(3)). Since we are looking for a fermionic content as close as possible to the Standard Model one, this choice for b must be forbidden. Let us point out that in supersymmetric models, extra doublets required by tadpole cancellation, could be interpreted as higgsinos and therefore, in such cases, there is no real need to have different U(2) behaviours for left quarks (if we allow for several Higgs fields). However, in this situation, $N = 3$ is needed in order to have three generations and this choice alway leads to extra matter. We conclude that it is *not possible* to obtain the, exactly, Minimal Supersymmetric Standard Model from a D-brane at an orbifold or orientifold singularity.

Several other restrictions must be imposed on a_α and N in order to avoid such kind of extra matter. For instance $a \neq -a, -b, -c$ in order to avoid $(\overline{3}, 2)$ states or $\frac{a-b}{2} + d_i \neq -a_\alpha$ to forbid anti-doublets etc. Similar constraints do appear when 73 sectors are included.

Observe that these restrictions are all *modulo* N and, therefore, will require N to be sufficiently large in order to forbidd identifications.

In fact, it appears, as we indicate below, that $N \geq 11$.

Let us see how all this works in explicit examples:

Consider the action on fermions given by odd twists a_α. Let us choose, for instance, $(a_1, a_2, a_3, a_4) = (1, 1, -5, 3)$. Hence, the corresponding action on scalars is achieved by $(b_1, b_2, b_3) = (-4, -4, 2)$.

The twist on Chan-Paton factors in (14) becomes $V = 1/N(\ldots, 2, 2, 2, 3, 3, \ldots)$ The constraints discussed above indicate already that $N \neq 2, 3, 4, 5, 6, 8$ or 10. $N = 7, 9$ can also be discarded by a more carefull analysis of 37 spectra. We will choose $N = 12$, for concreteness, and briefely discuss other possibilities afterwards.

A $\mathbb{Z}_{12}$ example:

Even if generic features are shared with other N's singularities, let us stress that $\mathbb{Z}_{12}$ is a peculiar example since it corresponds to a cristalographic singularity. Let us briefely comment about this. Classification of cristalographic singularities [14, 15] shows that only some $\mathbb{Z}_{12}$, non factorizable, singularities are possible and that factorizable one's are at most of $\mathbb{Z}_6$ type. Actually, notice that this corresponds to our case. Eventhough the action on fermions is given by a $\mathbb{Z}_{12}$ twist, the lattice is defined by the action on scalars which in the example at hand is given by $\frac{1}{12}(-4, -4, 2) = \frac{1}{6}(-2, -2, 1)$, namely, a product of three hexagonal (SU(3)) lattices. Therefore, it would be possible, for instance, to achieve a full consistent orientifold compactification In particular, the possibility of having large extra dimensions [16, 17] in order to lower the string scale, which is of phenomenological interest here since models are non supersymmetric, is open in this context.

Recall that $\sum b_i = even$ as is required by modular invariance in the closed, torus, sector. Nevertheless, since the singularity is not supersymmetric, tachyons in the closed string sector must be taken care of (see for instance [18])).

As explained above, in order to ensure twisted tadpole cancellation, we must first write down the generic spectrum and then find the conditions for it to be free of anomalies. After that we may choose specific values for the number of given CP twist eigenvalues ($n_i's, u_i's$ etc.) satisfying such requirements, in order to build a specific model.

Thus, for the 33 sector we define a generic twist

$$V_3 = \frac{1}{N}(1, \ldots, 1, 2, \ldots, 2, \ldots, 6, \ldots, 6)\,. \tag{15}$$

Where there are n_i entries equal to i on the D3-branes (n_0 is chosen to vanish) and similar vectors for $D7_1$,$D7_2$ and $D7_3$-branes with with u_i, v_i and w_i entries.

The spectrum reads

$$\begin{aligned}
&\text{Sector 33} \\
&2\left[(n_1,\overline{3}) + (3,\overline{2}) + (2,\overline{n}_4) + (n_4,\overline{n}_5) + (n_5,\overline{n}_6) + (n_5,n_6)\right] + \\
&+ (n_1,\overline{n}_4) + (3,\overline{n}_5) + (2,\overline{n}_6) + (2,n_6) + (n_4,n_5) + (\overline{n}_1,\overline{3}) + \\
&+ (\overline{n}_1,\overline{n}_6) + (\overline{n}_1,n_6) + (n_1,n_4) + (3,2) + (\overline{3},\overline{n}_5) + (\overline{2},\overline{n}_4)
\end{aligned} \tag{16}$$

$$\text{Sector } 37_3 \tag{17}$$
$$(n_1,\overline{w}_2)+(3,\overline{w}_3)+(2,\overline{w}_4)+(n_4,\overline{w}_5)+(n_5,\overline{w}_6)+(n_5,w_6)+$$
$$+(\overline{3},w_1)+(\overline{2},w_2)+(\overline{n}_4,w_3)+(\overline{n}_5,w_4)+(n_6,w_5)+(\overline{n}_6,w_5)$$

$$\text{Sector } 37_1 \tag{18}$$
$$(n_1,u_1)+(\overline{n}_5,\overline{u}_5)+(\overline{n}_4,\overline{u}_6)+(\overline{n}_4,u_6)+(n_6,\overline{u}_4)+(\overline{n}_6,\overline{u}_4)+$$
$$+(\overline{n}_1,u_3)+(\overline{3},u_4)+(\overline{2},u_5)+(2,\overline{u}_1)+(n_4,\overline{u}_2)+(n_5,\overline{u}_3)$$

and similarly for sector 37_2 by replacing $u \to v$. For the sake of clarity we have already chosen $n_2 = 3$ and $n_3 = 2$.

The twisted tadpole cancellation requirements read

$$-3n_1 + 3n_3 + w_3 - w_1 - u_4 - v_4 = 0 \tag{19}$$
$$-n2 + n_4 + 2n_6 + w_4 - w_2 + u_1 + v_1 - u_5 - v_5 = 0 \tag{20}$$
$$n_2 + 2n_4 - 2n_6 + w_2 + u_1 + v_1 - u_3 - v_3 = 0 \tag{21}$$
$$3n_5 - 3n_3 + w_5 - w_3 - 2u_6 - 2v_6 + u_2 + v_2 = 0 \tag{22}$$
$$-n_4 + 4n_6 - 2n_2 + 2w_6 - w_4 + u_3 + v_3 - u_5 - v_5 = 0 \tag{23}$$

Clearly $n_5 = n_4 = n_6 = w_2 = w_3 = u_5 = v_5 = 0$ in order to avoid extra triplets or anti-doublets and $n_1 = 1$. Interestingly enough, since $n_2 = 3$ and $n_3 = 2$, we find that first equation becomes $w_1 + u_4 + v_4 = 3$ telling us, as expected, that 3 anti-triplets must be provided by 37_r sectors. Similarly second equation indicates that $w_4 + u_1 + v_1 = 3$ doublets must come from such sectors. $3 + u_1 + v_1 = u_3 + v_3$ requires the total number of 1 and $\overline{1}$ to be the same. The other two equations come from cancellation of, generic, $\mathrm{SU}(n_4) \times \mathrm{SU}(n_5)$ anomalies even though here, $n_4 = n_5 = 0$.

By identifying the two anti-triplets $2(n_1, \overline{3})$ in 33 sector with U_R quarks (and therefore $(\overline{n}_1, \overline{3})$ with D_R) and placing the third one in 37_3 sector ($w_1 = 1$) we must define the hypercharge as

$$Y = \frac{Q_3}{6} - \frac{Q_1}{2} + \frac{Q_a}{2} + \frac{Q_b}{2} + \frac{Q_c}{2} + Q^7_{\text{neutral}}$$

with

$$Q_a = Q_4^{7_1} + Q_4^{7_2} - Q_1^{7_3} \tag{24}$$
$$Q_b = Q_4^{7_3} \tag{25}$$
$$Q_c = Q_3^{7_2} + Q_3^{7_1}$$

where the sub-indices indicate the corresponding Chan-Paton twist eigenvalue. We have summarized in Q^7_{neutral} the possibility of including other charges from 77 sectors. Even if SM massless fermions carry no such charge massless scalars could be charged.

Since the number of right handed leptons is given by $u_3 + v_3$, then $u_1 = v_1 = 0$ must be imposed.

We see that, due to the symmetry between 7_1 and 7_2 branes (see 8) there is still certain freedom to place some states in one or another sector leading to same spectrum.

Matter fields	Sector		Q_3	Q_2	Q_1	Q_a	Q_b	Q_c	Y
Q_L	(33)	$2(3,\overline{2})$	1	-1	0	0	0	0	1/6
q_L	(33)	$(3,2)$	1	1	0	0	0	0	1/6
U_R	(33)	$2(\bar{3},1)$	-1	0	1	0	0	0	$-2/3$
	(37_3)	$(\bar{3},1)$	-1	0	0	-1	0	0	$-2/3$
D_R	33	$(\bar{3},1)$	-1	0	-1	0	0	0	1/3
	37_2	$2(\bar{3},1)$	-1	0	0	1	0	0	1/3
L	37_3	$3(1,2)$	0	1	0	0	-1	0	$-1/2$
E_R	37_1	$3(1,1)$	0	0	-1	0	0	1	1

Table 1: Standard model spectrum and U(1) charges

As an example, let us choose to place three ($u_3 = 3$) right leptons in 37_1 sector and two D_R quarks ($v_4 = 2$) 37_2. Therefore, vector shifts

$$V_3 = \frac{1}{N}(1,2,2,2,3,3) \tag{26}$$

$$V_{7_1} = \frac{1}{N}(2,\ldots,3,3,3,6,\ldots,6) \tag{27}$$

$$V_{7_2} = \frac{1}{N}((2,\ldots,2,4,4,6,\ldots,6) \tag{28}$$

$$V_{7_3} = \frac{1}{N}(1,4,4,4,5\ldots,5,6,6,6) \tag{29}$$

with $w_5 + u_2 + v_2 = 6 + 2(u_6 + v_6)$, lead to $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$ spectrum

$$\begin{aligned} &2\left(3,\overline{2},\frac{1}{6}\right) + \left(3,2,\frac{1}{6}\right) + 3\left(\overline{3},1,-\frac{2}{3}\right) + 3\left(\overline{3},1,\frac{1}{3}\right) \\ &3\left(1,2,-\frac{1}{2}\right) + 3(1,1,1) \end{aligned} \tag{30}$$

Results are summarized in table 1

Notice that when above U(1) charges are global symmetries they can be given a familiar physical meaning (see [1] and [19] for similar observations). In particular, $Q_3 = 3B$ where B is the baryon number and $L = -Q_c - Q_b$ is the lepton number. Also $I_R = \frac{1}{2}(Q_1 - Q_a)$ corresponds to $\mathrm{SU}(2)_R$ weak isospin in left-right models etc. Recall, however, that at this level all such symmetries are actually local symmetries.

Several comments are in order. A relevant observation is that, correct hypercharge assignments, ensuring that above $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$ spectrum is anomaly free, unambiguously (up to some signs depending on which sector we chose to place right up or down quarks) define hypercharge Y above (24). Therefore, Y must involve generators of 77 sectors abelian groups.

As stressed before this appears to be an unavoidable fact also for other N's. There is no way to obtain the SM content from only the 33 sector without including extra matter.

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A consequence is that 7_r7_r states, which are singlets under $\mathrm{SU}(3)\times\mathrm{SU}(2)$, will generically carry hypercharge. Hence, mixed anomalies of Y with 7_r7_r groups should vanish in order for Y to be truly free of anomalies.

A complete study of this fact is out of the scope of this note where we are looking for a set of necessary conditions to achieve a *minimal* SM content. Notice, however, a somewhat related fact. Quite plausibly, Wilson lines (WL) could be introduced in order to break $\mathrm{U}(N)$ factors into products of abelian groups in 77 sectors (some of which will become massive). Thus, the dimension of 77 representations in 37 sectors will become true multiplicities for SM charged states as written down in eq. (30). Moreover, in this procedure, by suitably choosing WL, 77 states carrying hypercharge could be completely projected out of the spectrum thus leaving, at most, a certain number of SM singlets $(1,1,0)$ and/or hidden matter.

In order to indicate how this could work, let us turn on a discrete Wilson line on second complex plane [4, 8, 20]. This WL must be embedded as a twist on 7_1 and 7_3 branes. For instance, we can choose the twist on 7_1 -branes to be

$$W_{7_1}=\frac{1}{12}(3,\ldots,4,\ldots,5,\ldots,6,\ldots;0,1,2;3,\ldots) \tag{31}$$

where the first (last) entries correspond to $l_i\ i's\ i\neq 0,1,2$ ($m_i\ i's\ i\neq 0,1,2$) with $\sum l_i=u_2$ ($\sum m_i=u_6$). Such a WL breaks $\mathrm{U}(3)_3$ to $\mathrm{U}(1)^3$ and also projects out all Y charged states $(u_2,\overline{u}_3)_{-1/2}+(u_2,u_3)_{1/2}$ as desired. Notice that there is plenty of freedom for choosing WL producing such effect (moreover, in this particular case we could have chosen $u_2=0$ without even needing to introduce a Wilson line).

We can proceed similarly with the twist on 7_3 branes. The same goal can be achieved on 7_27_2 sectors by adding a WL on first (or third) plane with a corresponding action on 7_2 and 7_3 (7_2 and 7_1)-branes.

Unfortunately this is not the whole story. 7_r branes contain other four fixed points, apart from the origin. Twisted tadpoles are expected at such points and therefore extra D3'-branes must be placed there in order to cancel them. Again, among $3'7_r$ states, some of them will generically carry hypercharge. Since these are not fixed by the orientifold action, spectra and tadpole cancellation conditions will be of the form discussed in eq. (6–10). Moreover, such points will generically feel the presence of the Wilson lines.

Due to the freedom in choosing both Wilson lines and twists on D3'-branes we expect to be able to achieve such cancellation at each point, by forbidding (hyper)-charged $3'7_r$ states. For instance, fixed point $P=(0,+1,0)$ will feel a twist on 7_1 branes of the form

$$V_P=(V_{7_1}+W_{7_1},-V_{7_1}-W_{7_1})\,. \tag{32}$$

Tadpole cancellation at such point eq. (10) leads to twelve ($N=12$) equations. It can be checked that, in order to avoid (hyper) charged states $n'_1=n'_2=n'_3=n'_5=n'_6=n'_7=0$. However, even if this is very restrictive there is enough freedom in choosing n'_4, n'_8, n'_9, n'_{10}, n'_{11}, u_2 and u_6 WL entries above to solve the equations system.

Thus, by suitable introduction of Wilson lines, all (hyper)-charged 77 states could be projected out and relevant non-abelian groups broken down to abelian factors by leaving

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just gauge bosons and chiral fermions of the Standard Model (plus, presumably, SM singlets). Full consistent compactification will still require cancellation of untwisted tadpoles and twisted tadpoles proportional to inverse volume terms, not present in the infinite volume limit. In principle this could be achieved by following similar steps as in the compact models constructed in [4] and [21, 22] for example. Certainly anti-branes will be needed etc.

Here we have concentrated in the fermionic spectrum. However, above twists predict also the presence of scalars, as can be seen from (6) and 9, charged under SM group. For instance, scalars $2(3,1,-1/3)$ will always be present with a generic twist as in 14 in 33 sector etc.

In particular, the spectrum for the model presented above contains $2(3,1,-1/3)+3(1,2,1/2)$ in 33 sector and $(u_2+v_2)[(1,2,*)+(1,1,*)]+3(3,1,-1/3)+2(1,2,-1/2)$ from 37_1+37_2 sectors and $(1,2,1/2)+w_5[(1,1,*)+(1,2,*)]+6(3,1,*)$ from 37_3 sector. We have indicated by $*$ that Y charge depends on the definition of $Q^7_{\rm neutral}$ in (24).

We will not address a phenomenological study here. Let us say that we expect such scalars to become generically massive, since no symmetry that would keep them massless is operating here. They will contribute to the fermionic mass structure. Notice also that, among scalars, there are doublets with correct hypercharges to be identified with Higgs fields, which are required to break electroweak symmetry.

Twists defined above (26) correspond to a so called *with vector structure twist* [23] (roughly speaking $\gamma^N=1$). However, twists *without vector structure* ($\gamma^N=-1$) could also be considered. We have looked at $\mathbb{Z}_{12}$ examples with twist $a_\alpha \equiv 1/12(-1-1-24)$ (thus $b=1/12(-3,-3,-2)$) and generic CP embeddings $V_3=1/24(1,\ldots,1,3,\ldots 3,\ldots,11,\ldots,11)$. In such cases, twisted tadpole cancellation conditions appear to be much stronger than above (due essentially to the presence of antisymmetric representations, now allowed by α_α's) and, as a consequence, extra matter is required in order to satisfy them.

Let us conclude with a brief summary of our results and some observations. We have identified a set of generic necessary conditions which appear to be required if a *minimal* Standard Model content is looked for in the context of D-branes at $\mathbb{R}^6/\mathbb{Z}_N$ singularities. We have concluded that, $\mathbb{Z}_N$ must be an orientifold fixed point, N must be large enough ($N\geq 11$) and singularity must be non-supersymmetric. Moreover, we have shown that hypercharge involves combinations of U(1) generators which must include, necessarily, abelian factors from 77 sectors. We argued that extra 77 or 73' states carrying hypercharge could be projected out, for instance, by suitable introduction of Wilson lines. An explicit Z_{12} example was presented to illustrate such issues. Further examples, with higher order singularities, can be treated in the same way.[3] However, since such cases are not cristalographic, they should be embedded in a more complicated generic Calabi- Yau compact space.

Let us emphasize that this is just a first step in the construction of a fully consistent model. In particular, since singularities are non supersymmetric, closed string tachyons will be generically present. They could completely ruin the viability of these non-susy singularities or, more hopefully, they could be the signal of a transition towards more

[3]The spectrum can be obtained from 16–18 by a straightforward generalization taking care of mod N identifications.

stable configurations as in the situations analised in [24]. Let us notice that, depending on the θ action on scalars, (b_1, b_2, b_3), just closed twisted tachyons, associated to orbifold fixed points can be present or also closed tachyons propagating in the bulk (this is the case for the $\mathbb{Z}_{12}$ example above).

Another relevant point refers to the possible couplings of closed twisted RR fields to U(1) field strengths [12, 26]. Such couplings, through a generalized Green-Schwarz [25] mechanism, will ensure cancellation of U(1) charges combinations possessing triangle anomalies. Corresponding U(1) become massive. This should be the case, for instance, for Q_1 or Q_2 anomalous combinations.

Notice, however, that RR fields could couple to some (or all) anomaly free combinations, and render the corresponding gauge bosons massive. Further investigation of couplings is required in order to ensure that Y remains effectively a gauge symmetry. Assuming that this is indeed the case, let us recall that, for other U(1)'s becoming massive, original gauge symmetries remain as global symmetries [27] in the effective field theory. This leads to relevant phenomenological consequences. For instance, conservation of baryon number will protect proton from decaying etc. As stressed, we will not pursue a phenomenological study of models presented above. Nevertheless, notice that the kind of analysis made in ref [1] (see also [28]), in the context of D6-branes at angles referring to such global symmetries, can be paralleled here. In fact, this appears to be generic to SM built up from D-branes where originally U(n) factors rather than SU(n) are present.

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